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THE ORIGIN OF THE E-LAYER OF THE IONOSPHERE

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ABSTRACT

The electron concentration in the E-layer has been calculated as a function of height according to the theory of the pre-ionization of molecular oxygen by the sun's radiation (Nicolet) and subsequent dissociative recombination (Bates and Massey). The calculations are based on the recent work of Moses and Wu on the distribution of molecular and atomic oxygen in the atom-molecule transition region. It is shown that both the observed height and the value of the peak electron density can be accounted for satisfactorily. We also examine the photo-ionization of atomic oxygen, which leads to a peak in the region of the F_1-layer, but it does not seem possible to explain the observed effective recombination coefficient for the F_1-layer on the basis of theory involving atomic and molecular oxygen alone.

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I. Introduction

Since the proposal by Chapman of a theory of formation of ionized layers in the upper atmosphere by solar radiation, many attempts have been made to account for the observed properties of the various layers.[1] Recently it has been suggested rather more definitely than before that the E and F_1 layers arise mainly from the ionization of oxygen molecules and atoms respectively.[2] The arguments for these identifications are the following:

1. It is generally believed, on the basis of many theoretical studies, that the oxygen molecules in the upper atmosphere are dissociated into atoms by solar radiation, and that this dissociation takes place in a fairly sharp transition region, a few kilometers thick, which lies at altitudes between 90 and 110 Km. The details depend on the particular assumptions in the theory of the dissociation-recombination region. The E-layer is known to be situated at an altitude of about 100 Km[1], and on account of the dissociation there are very few oxygen molecules present above E-layer altitudes.

2. Of the various known chief constituents of the upper atmosphere, O , O_2 , N , N_2 , the oxygen molecule has the lowest ionization potential (12.2 ev, corresponding to a threshold wavelength of 1010 A). Hence only the radiations necessary to ionize O_2 (wavelengths immediately below 1010 A) are not absorbed by the overlying layers of the other constituents, and are thus able to penetrate down to the altitude of the E-layer.

3. On the other hand, the oxygen atoms extend to much greater heights than oxygen molecules, and it seems an attractive theory to identify the F_1 layer with the first ionization of the oxygen atom.

There are, however, difficulties in the above simple theory. The first of these is that the observed absorption cross section σ_2 leading to the ionization of O_2 in the first ionization limit (12.2 ev) [3]

$$O_2 + h\nu \longrightarrow O_2^+ + e; \; \sigma_2 \sim 10^{-19} \text{ cm}^2 \qquad (1)$$

is much too small to account for the observed height of the maximum electron density on the basis of the Chapman theory. It has been suggested by Nicolet[4] that the difficulty can be overcome by assuming that the O_2 absorbs radiation of wavelengths below 1010 A in transitions not leading to O_2^+ directly, but to excited states of O_2 that lie in the continuum above the first ionization limit of O_2 and converge to the second ionization limit of O_2 (i.e., to an excited state of O_2^+). The oxygen molecules in these excited states are in a condition for auto-ionization, so that the ionization of O_2 consists of two successive processes:

$$O_2 + h\nu \longrightarrow O_2^* \; ; \quad \lambda < 1010 \text{ A} \; ; \quad \sigma^* \tag{2}$$

$$O_2^* \longrightarrow O_2^+ + e \quad ; \quad p. \tag{3}$$

This suggestion is plausible, as the spectrum of O_2 has strong absorption bands in the region 910 - 1010 A (corresponding to 13.5-12.2 ev).[5] The cross section σ^* of the process (2) is not known, although a value of 4×10^{-17} cm^2 has been postulated arbitrarily[4]; and the transition probability p per sec. of the radiationless process (3) is also unknown. Let the transition probability (Einstein A coefficient) for the reverse process of (2) be denoted by A. Then the fraction of O_2^* produced by (2) which undergoes the auto-ionization process (3) is $p/(p + A)$, and the effective absorption cross section for O_2 leading to O_2^+ and e is

$$\sigma_2^{eff} = \frac{p}{p+A}\,\sigma^* + \sigma_2 . \tag{4}$$

While p is not known, it seems that $p < A$, since otherwise the absorption bands in (2) would have appeared diffuse by auto-ionization. In the present work we have tried a number of values of σ_2^{eff} which correspond to plausible values of σ^*, p, A, and shall show that values of σ_2^{eff} in this range are capable of accounting for the E-layer, when used in conjunction with the other data.

The second difficulty of the theory of the E and F_1 layers is that the observed recombination coefficients α for the E and F_1 layers have the values 10^{-8} and 4×10^{-9} cm^3 sec^{-1} respectively; these values are much larger than the coefficients for radiative recombination that can be deduced from the photo-ionization cross section[3,6] :

$$O_2^+ + e \longrightarrow O_2 + h\nu \qquad \alpha_{2,r} \sim 10^{-14} \text{ cm}^3 \text{ sec}^{-1} \tag{5}$$

$$O^+ + e \longrightarrow O + h\nu \qquad \alpha_{1,r} \sim 5 \times 10^{-13} \text{ cm}^3 \text{ sec}^{-1} \tag{6}$$

It has been suggested[2] that the O_2^+ may not recombine with electrons as in (5) but by the following " dissociative" recombination, which leads to two oxygen atoms in excited states:

$$O_2^+ + e \longrightarrow O' + O'' \; ; \quad \alpha_d \sim 10^{-8} \text{ cm}^3 \text{ sec}^{-1} , \tag{7}$$

and that O^+ and electrons may recombine by the two-stage process of a charge transfer:

$$O^+ + O_2 \longrightarrow O + O_2^+ ; \quad \gamma \sim 10^{-12} \text{ cm}^3 \text{ sec}^{-1} \tag{8}$$

followed by (7). The values of the coefficients α_2 and γ are not known either experimentally or theoretically; the values given above are assumed[2] on plausibility considerations to account for the observed α for the E and F_1 layers.

The purpose of the present work is to examine these theories of the E and F_1 layers by making a quantitative calculation of the electron densities resulting from the photo-ionization of molecular and atomic oxygen, on the basis of the recent work of Moses and Wu[7] on the distributions of O_2 and of O in and above the oxygen dissociation layer. While these distributions are not definitive on account of the lack of definite knowledge about the mechanism of association, they are, for each assumed model, soundly based on a "self-consistent" treatment, so that a calculation of the ionized layers using their results should be satisfactory in the same sense of " self-consistency".[8]

In the following, the number of particles X per cm^3 will be denoted by [X].

II. Photo-ionization of molecular oxygen and the E-layer

Recently it has been assumed[9] that the E-layer is due to the photo-ionization of oxygen atoms, involving the processes (6), (8) and (7). It will be shown in the next section, however, that the contribution to the electron density in the E-layer arising from oxygen atoms is extremely small, so that it is sufficient to assume here that the basic processes for the E-layer ionization are (2), (3), and (7).

The time rate of change of electron density is given by

$$d/dt\,[e] = q_2\,[O_2] - \alpha_d\,[O_2^+]\,[e], \tag{9}$$

where q_2 is the number of photo-ionizations per second, and is given by

$$q_2(x) = \int_{\nu_2}^{\infty} d\nu\, \frac{c}{h\nu}\, \rho_\nu(x)\, \sigma_2^{\text{eff}}(\nu), \tag{10}$$

Here x is the altitude, $\sigma_2^{\text{eff}}(\nu)$ is the effective cross section in (4), and $\rho_\nu(x)$, the energy density per unit frequency range at altitude x, is given by

$$\rho_\nu(x) = \rho_\nu(\infty)\, \exp\left\{ -\sigma_2^{\text{eff}}(\nu) \int_x^\infty [O_2]dx \right\}, \tag{11}$$

where $\rho_\nu(\infty)$ is the corresponding density at the top of the atmosphere. We shall make the usual assumption that $\rho_\nu(\infty)$ is the density of a black body at temperature Θ = 6000°K, "diluted" by a factor W, $4\pi W$ being the solid angle subtended by the earth at the sun. As the spectral regions concerned lie at or beyond 1000 A, it is sufficient to employ Wien's form for $\rho_\nu(\infty)$, so that

$$\rho_\nu(\infty) = \frac{8\pi W h \nu^3}{c^3} \exp(-\nu/\nu_s), \qquad \nu_s = k\Theta/h$$

$$W = 5.4 \times 10^{-6}. \tag{12}$$

Also, on account of the rapid decrease of ρ_ν with increasing frequency ν in this region, we may replace $\sigma_2^{\text{eff}}(\nu)$ by a constant equal to the value at the threshold ν_2. Thus (10) becomes

$$q_2(x) = \mathcal{Q} \exp\left\{ -\sigma_2^{\text{eff}} \int_x^\infty [O_2]\, dx \right\};$$

$$\mathcal{Q} = \frac{8\pi W}{c^2} \sigma_2^{\text{eff}} \nu_s^3 \; e^{-\xi_2}\left[\xi_2^2 + 2\xi_2 + 2\right], \tag{13}$$

where $\xi_2 = \nu_2/\nu_s$.

The condition for the ionosphere to be electrically neutral is that

$$[O_2^+] = [e]. \tag{14}$$

In order that equation (9), with (14), agree with the empirical relation in which the observed recombination coefficient is $\alpha = 10^{-8}$ cm^3 sec^{-1}, one assumes that

$$\alpha_d = \alpha = 10^{-8} \text{ cm}^3 \text{ sec}^{-1}. \tag{15}$$

The maximum electron density at noon, when

$$d/dt\,[e] = 0,$$

is given by

$$[e]^2 = q_2(\text{noon})\,[O_2]/\alpha_d, \tag{16}$$

a relation which holds for any height x. Thus, to obtain $[e]$ as a function of x, we need to know $[O_2]$ as a function of x, both for the factor $[O_2]$ in

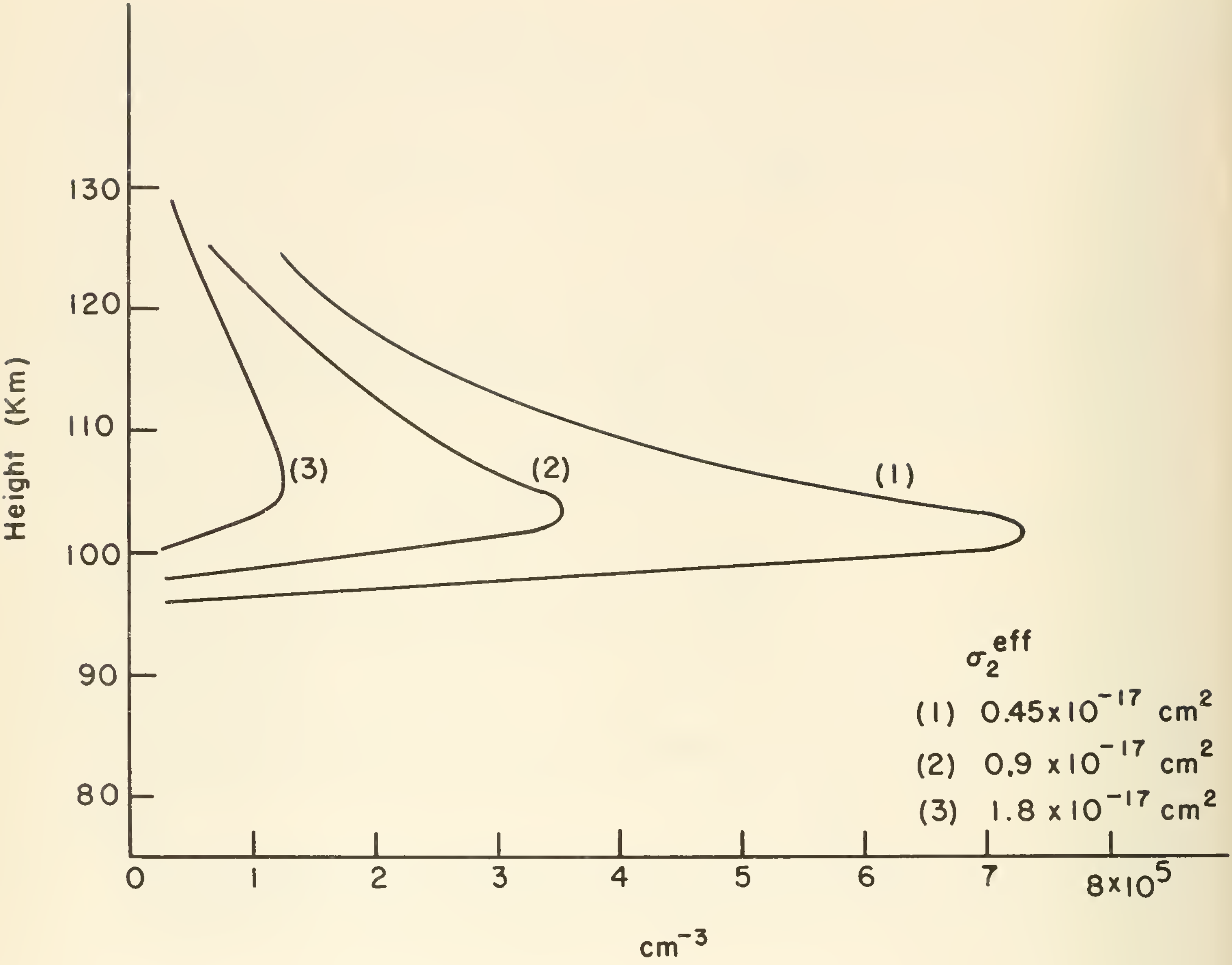

Figure 1

Concentration of free electrons [e] in the E-layer, resulting from the photo-ionization of molecular oxygen, for various values of the effective ionization cross section σ_2^{eff}.

The present curves result from the three-body model of Moses and Wu[7], with T=300°K and $C=10^{-32} cm^6 sec^{-1}$; the two-body model gives very similar curves, but shifted to lower altitudes by 8Km.

(16), and for the integrated O_2 - concentration that occurs in $q_2(x)$ from equations (10) and (11).

Detailed calculations have been carried out in the present work for two of the models studied by Moses and Wu[7], namely, their two-body recombination model with $T = 300^{\circ}K$, and their three-body recombination model with $T = 300^{\circ}K$ and a three-body recombination coefficient $C = 10^{-32}$ cm^6 sec^{-1}. Their other models (with different temperature distributions and other C values) give substantially similar results. The distributions of [e] as a function of x for the three-body model are given in Fig. 1 for various values of σ_2^{eff}, namely $\sigma_2^{eff} = 1.8 \times 10^{-17}$, 0.9×10^{-17}, 0.45×10^{-17} cm^2. With the two-body model, the curves obtained reproduce those in Fig. 1 to within 10% in electron concentration, but are shifted to about 8 Km lower.

The absolute magnitudes of [e] as given in Fig. 1 may be too large by as much as a factor of three, because $[e]^2$ is proportional to the energy density of solar radiation of the appropriate frequency (cf. equations (10)-(13), (16)). The energy density used in the computations corresponds to 12.2 ev; at 13.5 ev it is smaller by a factor of ten. From the results of Price and Collins[5] it is seen that there is a number of absorption bands in this frequency range, and their relative importance in contributing to the pre-ionization of O_2 is obviously very hard to evaluate. When allowance is made for this fact we see that the observed peak value $[e] = 1.5 \times 10^5$ cm^{-3} at a height of about 100 Km can be accounted for on the basis of the ionization of oxygen molecules.

III. Ionization of atomic oxygen and the F_1-layer

It is seen from the preceding section that for any reasonable distribution of molecular oxygen with height above the $O_2 \longleftrightarrow O$ dissociation region, the contribution from O_2 to the electron density decreases very rapidly with height above the E-layer, and is negligibly small at the height of the F_1-layer. On the other hand, the ionization of oxygen atoms is expected to give rise to an ionized layer with maximum electron density at greater altitudes than the E-layer. In the following, we shall again calculate the height of maximum electron density due to the ionization of atomic oxygen on the basis of the distributions of atomic oxygen given by the two models of Moses and Wu. It will be shown that, despite the qualitative agreement with the observed data, a complete understanding of the F_1-layer is still lacking.

The F_1-layer is characterized by the following observed data:

Height of maximum electron density = 200 Km

Peak electron density [e] = 2.5×10^5 cm^{-3} (17)

Recombination coefficient α = 4×10^{-9} cm^3 sec^{-1}.

The ionization process considered here is the photo-ionization of oxygen atoms just beyond the first ionization limit:

$$O + h\nu \longrightarrow O^+ + e \; ; \quad \lambda < 910 \text{ A} \; ; \quad \sigma_1 = 4.5 \times 10^{-18} \ \mathrm{cm}^2. \quad [6] \qquad (18)$$

As was mentioned in section I, the radiative recombination coefficient for the reverse process (6) is several orders of magnitude smaller than the observed α in (17). Hence, in any theory involving oxygen atoms and molecules alone, the only hope to account for the observed α is to assume that O^+ and e recombine through the two-stage process of charge transfer (8) followed by dissociative recombination (7). On this assumption the equations for the rate of change of [e] and $[O^+]$ are

$$d/dt \ [e] = q_1(x)[O] - \alpha_d \, [O_2^+] \, [e] \qquad (19)$$

$$d/dt \ [O^+] = q_1(x)[O] - \gamma \, [O^+] \, [O_2], \qquad (20)$$

where $q_1(x)$ is similar to $q_2(x)$ of equation (10), but always with oxygen atoms instead of oxygen molecules, and with σ_1 instead of σ_2^{eff}, so that

$$q_1(x) = \int_{\nu_1}^{\infty} d\nu \ \frac{c}{h\nu} \ \rho_\nu(x) \ {}_1(\nu) \qquad (21)$$

$$\rho_\nu(x) = \rho_\nu(\infty) \ \exp\left\{ - \sigma_1(\nu) \int_x^{\infty} [O] \, dx \right\}. \qquad (22)$$

The condition for the F_1-layer to be electrically neutral is

$$[O^+] + [O_2^+] = [e] \qquad (23)$$

so that (19) becomes

$$d/dt \ [e] = q_1(x) \ [O] - \alpha_d \left\{ 1 - \frac{[O^+]}{[e]} \right\} \ [e]. \qquad (24)$$

At noon,

$$d/dt\,[e] = d/dt\,[O^+] = 0,$$

and it can be shown by means of (19), (20), (23) that the electron density is given by either of the following equivalent expressions:

$$[e]^2 = \frac{q_1\,(\text{noon})\,[O]}{\alpha_d}\left\{1 - \frac{q_1\,[O]}{\gamma\,[O_2][e]}\right\}^{-1} \tag{25a}$$

$$[e]^2 = \frac{q_1\,(\text{noon})\,[O]}{\alpha_d\,\gamma\,[O_2]}\left\{\alpha\,[e] + \gamma\,[O_2]\right\} \tag{25b}$$

At E-layer altitudes, $\gamma\,[O_2] \sim 10^{-12} \times 10^{13}$ $\sec^{-1}$, and $\alpha_d\,[e] \sim 10^{-8} \times 10^{5} \sec^{-1}$, so that the effective recombination coefficient

$$\alpha_d\,[O_2] \Big/ \left\{\alpha_d\,[e] + \gamma\,[O_2]\right\}$$

is practically α_d. Calculations of $q_1(x)\,[O]\,(x)$ on the basis of the distributions of atomic oxygen in any of the models of Moses and Wu[7] (or of other authors for that matter) show that the contribution to [e] from the photoionization of oxygen atoms is negligibly small in the E-layer (cf. table 1). This seems to have eliminated atomic oxygen as an important source of electrons in the E-layer.[9]

To calculate [e] in (25) on the basis of the models of Moses and Wu[7], one has to extend the concentrations of atomic and molecular oxygen given by Moses and Wu above the $O_2 \longleftrightarrow O$ transition region. In these altitudes ($x > 130$ Km) the distribution of O is governed by diffusion rather than by dissociation-association. Instead of assuming an isothermal atmosphere, it is perhaps in better accord with empirical facts to assume a linear temperature rise with height;

$$T(x) = T(x_E) + b(x - x_E)\ , \tag{26}$$

where x is taken as some convenient point near the top of the E-layer; the most reliable values of b that have been deduced are 2.15×10^{-5} deg K/cm for an atmosphere of variable molecular weight, or 5.53×10^{-5} for constant molecular weight of 28.966[10].

For an atmosphere with the temperature distribution (26), the density of a species X varies as

$$[X] = [X]_E \left\{ 1 + \frac{b(x-x_E)}{T(x_E)} \right\}^{-\left(1 + \frac{M_X g}{kb}\right)} \tag{27}$$

We shall calculate $[e]$ from the empirical relation

$$\alpha [e]^2 = q_1(x) [O] \tag{28}$$

in which α has the constant empirical value 4×10^{-9} cm^3 sec^{-1}. The results of a number of models are given in table I. It is seen that if one uses temperatures derived from the assumption of variable molecular weight (case (1) in the table), one arrives at results in passable agreement with the experimental data[1] (17), and one is tempted to identify the F_1-layer with the region of ionization of oxygen atoms.

The above result, however, is obtained by using the empirical relation (28) instead of (25). When the "theoretical" relation (25a) is employed for heights near the F_1-layer, one obtains the meaningless result that $[e]^2$ becomes negative as $q_1 [O]$ becomes greater than $\gamma [O_2] [e]$. Alternatively, one may impose the empirical value 4×10^{-9} cm^3 sec^{-1} on the theoretical expression for α in (25b):

$$\frac{\alpha_d \gamma [O_2]}{\alpha_d [e] + \gamma [O_2]} = 4 \times 10^{-9} \text{ cm}^3 \text{ sec}^{-1}, \tag{29}$$

and one finds the necessary value for $[O_2]$:

$$[O_2] \sim \frac{2}{3} \frac{\alpha_d [e]}{\gamma} = \frac{2}{3} \times 10^9 \text{ cm}^{-3},$$

which corresponds to a much lower altitude than the F_1-layer.

The above difficulty is not resolved by including the radiative recombination (6) for the F_1-layer. In this case, an additional term $-\alpha_{1,r}[O^+][e]$ is to be added to the right-hand sides of equations (19) and (20), and the maximum electron density at noon is given by

$$[e]^2 = \frac{q_1 [O]}{\alpha_d} \frac{\alpha_d [e] + \gamma [O_2]}{\alpha_{1,r} [e] + \gamma [O_2]} \tag{30}$$

instead of (25). This relation obviously meets with the same difficulty as (25b). In fact, it can readily be shown by similar arguments that it is not possible to have a complete theory of the F-layer (especially of the observed value $\alpha = 4 \times 10^{-9}$ $cm^3\ sec^{-1}$) based on processes involving oxygen atoms and molecules alone, such as (2), (3), (5), (6), (7), (8), (18). The difficulty of accounting for the observed α for the F_1-layer lies in the smallness of the concentration of O_2 in that region, and the smallness of the radiative recombination coefficient $\alpha_{1,r}$ of (6). The rapid decrease of $[O_2]$ with height above the E-layer as a result of the dissociation of molecular oxygen into atoms renders such processes as (8) and (7) ineffective.

This difficulty of explaining the observed recombination coefficient of the F_1-layer in terms of oxygen atom ionization is rather independent of the particular theory for the oxygen distribution employed in the calculation. Furthermore, the difficulty is not resolved by adding the contribution from the ionization of nitrogen molecules at the first ionization potential of 15.5 ev [2] to the contribution from atomic oxygen. Thus, on assuming

$$N_2 + h\nu \longrightarrow N_2^+ + e \quad ; \quad \sigma_N = 10^{-17}\ cm^2 \text{ at } 15.5 \text{ ev}^{[2]} \tag{31}$$

$$N_2^+ + e \longrightarrow N' + N'' \quad ; \quad \alpha_N \sim 4 \times 10^{-9}\ cm^3\ sec^{-1} \tag{32}$$

in addition to (18), (7), (8), it can be shown that the maximum electron density at noon is given by

$$[e]^2 = \frac{q_1[O]}{\alpha_{1,r}} + \frac{q_2[N_2]}{\alpha_N} \tag{33}$$

and the effective recombination coefficients is given by

$$\alpha_r \left\{ 1 + \frac{q_N[N_2]}{q_1[O]} \right\} \frac{[O^+]}{[e]} = \alpha_N \left\{ 1 + \frac{q_1[O]}{q_N[N_2]} \right\} \frac{[N_2^+]}{[e]} .$$

Calculations of $q_N[N_2]$, $q_1[O]$ at the heights of the F_1-layer (200 Km) based on the three-body association model of Moses and Wu[7] and on the assumption of a linear rise of temperature with altitude between 100 and 200 Km, as in equation (26), show that

$$\frac{q_N[N_2]}{q_1[O]} \sim 1/160 \qquad \text{at } 200 \text{ Km.}$$

This means that at this height the contribution to the electron density comes predominantly from the oxygen atoms, and the rate of recombination is determined essentially by that between O^+ and electrons.

Table I. Photo-ionization of atomic oxygen.

		Two-body model $T = 300^\circ K$		Three-body model $T = 300^\circ K$ $C = 10^{-32}\ cm^6\ sec^{-1}$	
Situation near the top of the E-layer at height x_E					
x_E	(Km)	97		105.5	
$[O]_E$	(cm^{-3})	3.86×10^{13}		4.3×10^{12}	
$[O_2]_E$	(cm^{-3})	2.32×10^{11}		1.67×10^{11}	
$[e]_E$	(cm^{-3})	$\ll 1$		~ 1	
Maximum in the electron concentration due to photo-ionization of atomic oxygen					
		(1)	(2)	(1)	(2)
height of maximum	(Km)	220	305	170	190
α (empirical[1])	($cm^3\ sec^{-1}$)	4×10^{-9}	8×10^{-11}	4×10^{-9}	4×10^{-9}
$[e]$	(cm^{-3})	1.5×10^5	6.6×10^5	1.7×10^5	1.3×10^5

(1) refers to temperatures calculated by using variable molecular weight [10]. thus in equation (26), $b = 2.15 \times 10^{-5}$ degK/cm.

(2) refers to temperatures calculated for fixed molecular weight $M = 28.966$ ($b = 5.53 \times 10^{-5}$ degK/cm). The effective molecular weight is certainly not constant at these altitudes, and thus (1) should be considerably more reliable than (2).

REFERENCES

[1] cf. S. K. Mitra, The Upper Atmosphere, 2nd. Ed. (1952), Calcutta; pp. 290-291.

[2] D.R. Bates and H.S. W. Massey, Proc. Roy. Soc. A187, 261 (1946); A192, 1 (1947); H. S. W. Massey, Phil. Mag. Supplement 1,395 (1952), section 5.

[3] K. Watanabe, F. Marmo, E.C.Y. Inn, Phys. Rev. 90,155 (1953).

[4] M. Nicolet, Mem. Roy. Met. Inst. Belg., 19 (1945).

[5] W.C. Price and G. Collins, Phys. Rev. 48,714 (1935).

[6] T. Yamanouchi and M. Kotani, Proc. Phys.-Math. Soc. Japan, 22,60 (1940).

[7] H.E. Moses and T.Y. Wu, Phys. Rev. 87,628 (1952) for the "two-body" association model; and result, to be published, for the "three-body" association model.

[8] D.C. Choudhury, Phys. Rev. 88,405 (1952), has calculated the absorption cross section σ_2^{eff} needed for (2) and (3) in order to account for the observed height of maximum electron density in the E-layer according to the simple Chapman theory, on the basis of the earlier work of Moses and Wu, Phys. Rev. 83,109 (1951). It is preferable, however, to replace that treatment of Moses and Wu by their later work mentioned in ref. 7.

[9] E. Gerjuoy and M.A. Biondi, Westinghouse Research Paper No. 1708, Oct. 1952. We should like to thank these authors for letting us see their work before publication.

[10] The Rocket Panel, Phys. Rev. 88,1027 (1952).